# HORIZONTAL PARTICLE MIXING IN A FREE FLUIDIZED BED

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Horizontal particle diffusion in free fluidized beds is investigated experimentally.

The intensity of mixing of the solid phase over the volume of a fluidized bed has a considerable influence on the mass-transfer processes taking place in it. In this connection there is an obvious need to know both the basic laws of particle mixing and the values of the parameters of theoretical models of the process.

It is now standard [1] to describe horizontal particle mixing in fluidized beds by the diffusional model

$$\frac{\partial c}{\partial t} = D_{ef}^{h} \frac{\partial^2 c}{\partial y^2} . \qquad (1)$$

In this connection, the problem of studying horizontal mixing is reduced to the experimental determination of the single parameter of the model described by (1), the effective coefficient of horizontal particle diffusion ($D_{ef}^{h}$).

In the literature there are quite a lot of disconnected experimental data [2-10] on the values of $D_{ef}^{h}$ found by comparing experimental mixing curves with solutions of Eq. (1) under the corresponding boundary conditions. The results of individual investigations are often contradictory and there is no unified opinion on the character of the influence of various parameters of a fluidized bed on the intensity of horizontal particle transfer – on the value of the coefficient $D_{ef}^{h}$. This pertains primarily to the dependence of $D_{ef}^{h}$ on the height of the bed and the cross-sectional area of the apparatus. The solution to this problem is directly connected with the general problem of scale conversion, consisting in the determination of the conditions when it is justified to use data obtained on small laboratory installations in the design and exploitation of large industrial apparatus. As a consequence, there is now an absence of sufficiently universal correlations for calculating $D_{ef}^{h}$ which are valid in a wide range of experimental conditions.

In the present report an attempt is made, on the basis of our experimental investigation of the coefficient $D_{ef}^{h}$ and literature data, to obtain such a correlation, to establish the degree of influence of the geometrical size of the system on the intensity of horizontal particle mixing, and thereby to clarify one aspect of the problem of scale conversion in fluidized beds.

The tests were conducted in four different apparatus (Table 1). The characteristics of the disperse materials investigated are given in Table 2. A description of the test stand with a column diameter of 70 cm is given in [11]. The setups with the other apparatus were similar. In all the problems the gas distributors consisted of two perforated plates between which layers of heavy cloth were pressed, and they provided sufficiently uniform gas distribution.

The coefficient of horizontal particle diffusion $D_{ef}^{h}$ (the coefficient of horizontal thermal diffusivity of the bed) was found by the nonsteady method by a procedure which allows for heat removal from the bed by the filtering air and which is presented in [12] in application to tests in rectangular apparatus. To calculate tests in the round column C.1 we solved the boundary problem

$$\frac{\partial \theta}{\partial \mathrm{Fo}} = \frac{\partial^2 \theta}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial \theta}{\partial \xi} - \mathrm{Pe}\,\theta;\ \theta(\mathrm{Fo}, 0) < \infty;$$

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TABLE 1. Experimental Conditions

| Disperse material (see Table 2 | Column (designation in text) | $H_0$, cm | u, cm/sec |
|---|---|---|---|
| P. 1 | $D_c$ = 70 cm (C. 1) | 12,5; 25,5; 40,0 | 8,5—47,5 |
| P. 3 | S = 20×60 cm$^2$ (C. 2) | 5,5; 13,0; 23,0; 35,0 | 37,0—125,0 |
| P. 3 | S = 25×40 cm$^2$ (C. 3) | 14,5; 23,0; 35,3 | 26,0—89,0 |
| P. 4 | | 23,7; 37,5 | 69,0—141,0 |
| P. 5 | | 13,0 | 58,0—97,0 |
| P. 2 | S = 5×50 cm$^2$ (C. 4) | 14,0; 20,0 | 8,0—15,5 |

TABLE 2. Characteristics of Disperse Materials Investigated

| Disperse material | Designation in text | d, mm | $u_0$, cm/sec | $\rho_s$, g/cm$^3$ | $\delta/d$ |
|---|---|---|---|---|---|
| Quartz sand (wide fraction) | P. 1 | 0,24 | 5,0 | 2,65 | 0,24 |
| Quartz sand (narrow fraction, 0.200-0.315 screen) | P. 2 | 0,26 | 5,5 | 2,65 | — |
| Quartz sand | P. 3 | 0,60 | 20,0 | 2,54 | 0,22 |
| Glass beads | P. 4 | 1,75 | 63 | 2,60 | 0,20 |
| Millet | P. 5 | 2,0 | 55 | 1,02 | — |

$$\theta(0,\ \xi) = \begin{cases} 1, & 0 \leqslant \xi < R \\ 0.5, & \xi = R \\ 0, & R < \xi \leqslant 1 \end{cases}; \quad \frac{\partial\theta(\mathrm{Fo},\ 0)}{\partial\xi} = \frac{\partial\theta(\mathrm{Fo},\ 1)}{\partial\xi} = 0. \tag{2}$$

The solution of (2) was obtained by the method of separation of variables [13] and has the form

$$\theta = \exp(-\mathrm{PeFo})\left[R^2 + 2R\sum_{n=1}^{\infty}\frac{1}{\mu_n}\frac{J_1(\mu_n R)}{J_0^2(\mu_n)}J_0(\mu_n\xi)\exp(-\mu_n^2\mathrm{Fo})\right], \tag{3}$$

where $J_0$ and $J_1$ are zeroth- and first-order Bessel functions of the first kind; $\mu_n$ are roots of the function $J_1$ ($\mu_1$ = 3.83; ...). From (3) we calculated the dependences $\theta$(Fo) for different values of Pe for the cylindrical surface $\xi = \xi_0 = 0.93$ (the mounting point of the measuring thermocouples, which was chosen from construction considerations). This allowed us to construct the function $\mathrm{Fo}^{\max} = 4D_{ef}^h\tau_{\max}/D_c^2 = f(\mathrm{Pe})$, which determines the time dependence of the onset of the temperature maximum ($\tau_{\max}$) on Pe, characterizing heat removal from the bed with the departing air. The coefficient $D_{ef}^h$ was found from the experimentally measured values of $\tau_{\max}$ and the theoretical dependence $\mathrm{Fo}^{\max} = f(\mathrm{Pe})$ by the method of successive approximations given in [12].

The experimental procedure consisted in the following. The bed was fluidized by air at room temperature with a given filtration velocity. A movable barrier (a plate in the rectangular apparatus and a thin-walled pipe in the cylindrical apparatus) was inserted into the apparatus, dividing the system into two parts, a heating chamber ($0 \leqslant \xi < R$) and a working chamber ($R < \xi \leqslant 1$). The values of R in the different apparatus were 0.1-0.15. A small portion (less than 1% of the volume of the bed) of material preheated to 400-600°C was poured into the heating chamber. After a certain time, required for mixing of the hot particles in the heating chamber (about 5-10 sec), the movable barrier was rapidly removed (in not more than 0.5 sec) from the bed and the bed temperature ($T_s$) averaged over the vertical surface $\xi = \xi_0$ was measured with time by the system of thermocouples. Depending on the height of the bed, we used from three to nine thermocouples connected in parallel, distributed uniformly in vertical plates $\xi_0 = 1$ (column C. 2), $\xi_0 = 0.5$ and 1 (column C. 3), $\xi_0 = 0.9$ (column C. 4), and at the cylindrical surface $\xi_0 = 0.93$ (column C. 1). The emf of the thermocouples was fed through the amplifier of a V2-15 microvoltmeter to an N327-3 recorder, which provides continuous tape-recording of the signal. From three to six independent tests were conducted for each fluidization mode.

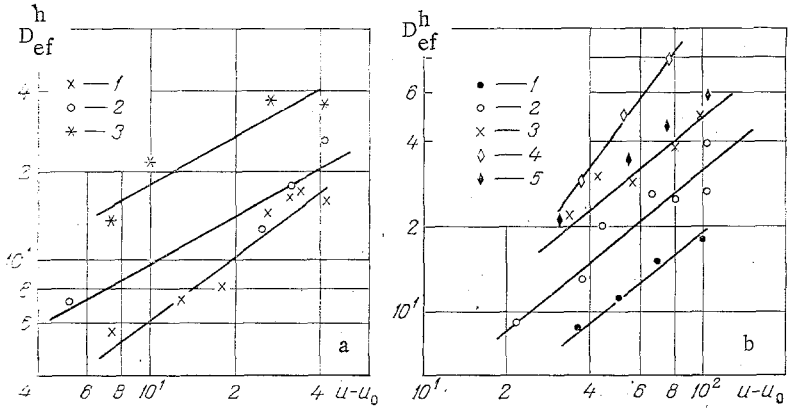


Fig. 1. Dependence of the effective coefficient of horizontal particle diffusion on the excess air filtration velocity: a: 1, 2, 3) $H_0$ = 12.5, 25.5, 40 cm (column C. 1); b: 1-4) $H_0$ = 5.5, 13, 23, 35 cm (column C. 2); 5) $H_0$ = 23.7 cm (material P. 4, column C. 3). $D^h_{ef}$, $cm^2/sec$; $u - u_0$, cm/sec.

Some results are given in the figures. They indicate the significant increase in $D^h_{ef}$ with an increase in the initial bed height and the excess air filtration velocity. The particle size had no noticeable effect on the coefficient $D^h_{ef}$ (Fig. 1, b).

To generalize the data obtained and those available in the literature on the values of $D^h_{ef}$, we must consider the mechanism of horizontal mixing of the solid phase. As is known [14], from the aspect of the height distribution of porosity in the bed it is divided into three zones: a) a grid zone (with an increased porosity which varies with height); b) the core of the bed (a constant porosity); c) a zone of breaking of gas bubbles (the porosity increases to one with height). These same regions are also distinguished by different mechanisms of horizontal particle mixing, as the observations show. In zones a) and c) the motion of the solid phase has a directional (horizontal) character: Closure of the vertical circulation loops of zone b) takes place in them; the core of the bed is characterized by a turbulent mixing mechanism due to particle flow around gas bubbles and the horizontal particle motions which develop in the process. The contributions of different zones to the total horizontal particle transfer (and that averaged over the bed height) are not proportional to the sizes of these regions, generally speaking, owing to the different character of the particle motion in them, and they have not been studied in detail in the literature. The statement made in [15] that the main horizontal transfer of particles (and heat) is provided by zones a) and c) does not seem universal and is evidently valid only in low beds. Considering the fact that horizontal particle transfer in all three zones is dependent on the passage of gas bubbles through the bed, it is natural to connect $D^h_{ef}$ with their characteristics. It seems consistent to do this in the approximation of turbulent diffusion, which agrees with the model adopted for the description of horizontal mixing (1).

In accordance with this assumption, the effective coefficient of turbulent diffusion is defined as [16]

$$D^h_{ef} = kL\bar{v}, \tag{4}$$

where L is the primary scale of the turbulent pulsations or the mixing path of a turbulent "vortex"; $\bar{v}$, average velocity of turbulent pulsations; k, proportionality factor. Since "pseudoturbulent vortices" in a bed are formed by the passage of gas bubbles, it is natural to set $L = \frac{1}{H}\int_0^H D_b dx$; $\bar{v} = \frac{1}{H}\int_0^H u_b dx$. Using the experimental correlations established in [17] for calculating $D_b$ and $u_b$, from (4) we obtain

$$D^h_{ef}/(u - u_0)H_0 = k_1\hat{p}^2(D_c/H_0)^{0.5}. \tag{5}$$

The test data (Fig. 1) showed that $D^h_{ef}$ does not grow linearly with an increase in $u - u_0$, rather $D^h_{ef} \sim (u - u_0)^{0.7-0.8}$. With allowance for this, we introduced into (5) the Froude number $Fr = (u - u_0)^2/gH_0$, characterizing the ratio between the kinetic and potential energy of a gas bubble and successfully used earlier to generalize data on longitudinal particle

TABLE 3. Experimental Conditions for Determining the Coefficient of Horizontal Particle Diffusion

| Literature | Material | $u_0$, cm/sec | d, mm | Column | $H_0$, cm | u, cm/sec | Measurement method |
|---|---|---|---|---|---|---|---|
| [3] | Polyvinyl chloride | 25 | 0,6 | $S=30\times90$ cm$^2$ | 17; 24; 27; 31 | 33—71 | Washing out a colored tracer |
| [4] | Quartz sand | 75 | 1,5 | $S=50\times250$ cm$^2$ | 3,25 | 60—320 | Instantaneous heat pulse |
| [5] | Copper–nickel shot | 1,64 | 0,096 | $S=4,6\times17,8$ cm$^2$ | 1,77; 3,54; 5,30 | 15,2—46,2 | Washing out a magnetic tracer |
| [6] | Millet | 60 | 2,0 | $S=10\times100$ cm$^2$ | 4,5; 8,5; 9,5 | 120—216 | Washing out a colored tracer |
| [6] | Limestone<br>Polystyrene | 62<br>76 | 2,5<br>3,0 | $S=10\times100$ cm$^2$ | 6,0<br>6,0 | 207<br>152—342 | Same |
| [7] | Aluminum oxide | 6,0 | 0,09 | $S=7\times100$ cm$^2$ | 8; 15; 37 | 14—38 | Stationary thermal mode |
| [8] | Sand | 10 | 0,35 | $D_c=40$ cm$^2$ | 6; 12 | 22,5; 46,0 | Washing out a radioactive tracer |
| [9] | Corundum | 5<br>20 | 0,12<br>0,32 | $S=20\times80$ cm$^2$ | 18<br>18 | 12÷40<br>27÷55 | Instantaneous heat pulse |

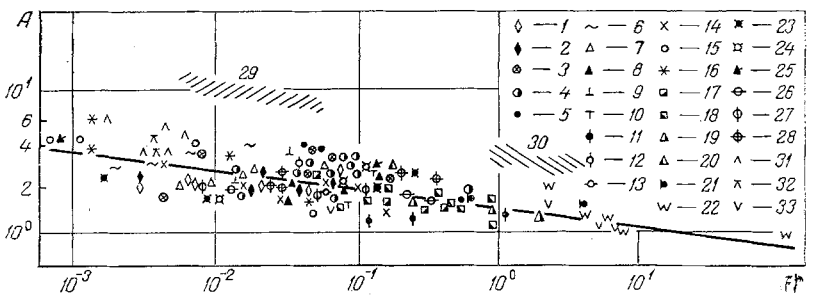


Fig. 2. Generalization of experimental data on effective radial particle diffusion in free fluidized beds (see Table 3): 1, 2) [9] (d = 0.12 and 0.32 mm); 3-5) [3] ($H_0$ = 17, 24, 27.5 cm); 6-8) [7] ($H_0$ = 37, 15, 8 cm); 9-10) [8] ($H_0$ = 12.6 cm); 11-13) [5] ($H_0$ = 1.77, 5.3, 3.54 cm); 14-16) (C. 1; $H_0$ = 12.5, 25.5, 40 cm); 17-20) (C. 2; $H_0$ = 23, 13, 5.5, 35 cm); 21, 22) [6] (limestone and polystyrene, $H_0$ = 6 cm); 23, 24) (C. 3; P. 4; $H_0$ = 23.7, 37.5 cm); 25) (C. 3; P. 5; $H_0$ = 13 cm); 26-28) (C. 3; P. 3; $H_0$ = 23, 35.5, 14.5 cm); 29) [3] ($H_0$ = 31 cm); 30) [6] (millet, $H_0$ = 4.5, 9.5, 8.5 cm); 31, 32) (C. 4; $H_0$ = 20, 14 cm); 33) [4]. $A \equiv [D^h_{ef}/(u-u_0)H_0](H_0/D_c)^{0.5}\cdot10^2$.

mixing in free and organized beds [17]. Data (Table 3) on the values of the coefficient $D^h_{ef}$ obtained and the literature data [3-9] were analyzed by the method of least squares through the dependence

$$D_{ef}/(u-u_0)H_0 = k_2(D_c/H_0)^{n_1}\mathrm{Fr}^{n_2}. \quad (6)$$

We excluded from the analysis the data of [10], where an unexpected and rather strange result was obtained: A decrease in the horizontal mixing of corundum particles (d = 0.12, 0.32 mm) with an increase in the initial bed height. This may be explained by an error which arose from ignoring the influence of heat removal from the bed with the departing air on the value of $\tau_{max}$ in the adopted method of analysis of the test data. For the coefficients $k_2$, $n_1$, and $n_2$ we obtained $k_2$ = 0.013, $n_1$ = 0.5, and $n_2$ = —0.15. The experimental values of $D^h_{ef}$ repre-

sented in accordance with (6) are shown in Fig. 2. As $D_c$ for the rectangular apparatus we chose $D_c^{equ} = \sqrt{4S/\pi}$. The rms deviation of the test data from (6) is 32%. The data of [3] for $H_0$ = 31 cm and of [6] for polished millet were separated from this set of points described by (6). The dependence (6) was obtained in the following ranges of the determining parameters: $6 \leqslant Fr \leqslant 8 \cdot 10^4$; $0{,}81 \leqslant D_c/H_0 \leqslant 39$; $10{,}2 \leqslant D_c \leqslant 70$ cm; $7 \leqslant u - u_0 \leqslant 157$ cm/sec; $1{,}77 \leqslant H_0 \leqslant 40$ cm; $1 < u/u_0 \leqslant 25$; $1{,}64 \leqslant u_0 \leqslant 76$ cm/sec. Equation (6) gives the dependence of $D_{ef}^h$ on the parameters of a fluidized bed affecting the horizontal particle mixing in the form $D_{ef}^h \sim D_c^{0{\cdot}5} H_0^{0{\cdot}65} (u - u_0)^{0{\cdot}7}$.

The generalized correlation (6) is valid in a wide range of experimental conditions and allows for the influence of the scaling factor on the effective coefficient of horizontal diffusion of the solid phase. It can be used in calculations of industrial equipment containing fluidized beds under conditions of uniform gas distribution.

## NOTATION

$a_{ef}^h$, effective coefficient of horizontal thermal diffusivity of bed; $c_f$, $c_s$, specific heats of gas and particles at constant pressure; c, concentration of tracer particles; $D_b$, diameter of a gas bubble; $D_c$, column diameter; $D_{ef}^h$, effective coefficient of horizontal particle diffusion ($D_{ef}^h = a_{ef}^h$); d, average particle diameter; g, free-fall acceleration; $H_0$, H, bed heights at filtration velocities $u_0$ and u, respectively; $l$, bed width; $\hat{p} = H/H_0$, bed expansion; r, radial coordinate; R, relative width of heating chamber; S, cross-sectional area of apparatus; $T_c$, $T_0$, temperature of heating chamber and air inlet temperature; $T_s$, bed temperature; $u_b$, ascent velocity of a gas bubble; $u_0$, u, velocity of onset of fluidization and filtration velocity; x, y, vertical and horizontal coordinates; $\delta$, dispersion of particle size distribution function; $\varepsilon_0$, porosity of emulsion phase of bed; $\theta = (T_s - T_0)/(T_c - T_0)$, dimensionless bed temperature; $\xi = 2r/D_c$ (round column); $\xi = y/l$ (rectangular column); $\rho_f$, $\rho_s$, gas and particle densities, respectively; t, time; $\tau_{max}$, time of onset of the temperature maximum at a chosen point of the bed; $Fo = 4D_{ef}^h t/D_c^2$, Fourier number; $Fr = (u - u_0)^2/gH_0$, Froude number; $Pe = [\rho_f c_f u D_c^2] : [4\rho_s (1 - \varepsilon_0) c_s D_{ef}^h H_0]$, Peclet number.

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# MASS TRANSFER IN THE MECHANICAL MIXING OF HETEROGENEOUS SYSTEMS

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The influence of mixing on the kinetics and intensity of mass transfer is examined for dissolution and extraction in fluid-solid systems.

Among the mixing and mass transfer processes in apparatus with agitators, the phenomena of mass transport in heterogeneous fluid–solid systems are greatly widespread. However, hydrodynamics investigations only touch upon the "equilibrium" distribution of solid particles over the height of the vessel [1]. Mass transfer questions are also studied inadequately, and are limited to an analysis of the mass elimination coefficients during mixing under stationary conditions [2]. Practice shows that the duration of such processes as extraction or dissolution can be sufficiently large [3]; consequently, it is necessary to know the kinetic regularities of mass transfer during mixing. An attempt is made below to develop a model of mass transfer processes for the mechanical mixing of heterogeneous systems that would permit the investigation of the transport kinetics of solid particles and their mass elimination to the carrying fluid phase, with the change in particle size during dissolution, the system properties, and the geometric characteristics and operating conditions of the equipment taken into account.

Earlier we examined the hydrodynamics models [4] and mixing processes [5] of homogeneous systems. It was shown that the application of cellular circulation material transport models with a computation of the fluxes through the cell on the basis of an analytic spatial model of the hydrodynamics of apparatus with agitators will permit sufficiently efficient investigation of the dynamics of macromixing processes for liquid media in industrial apparatus. The apparatus working space can, when modeling the mixing of fluid–solid systems, also be divided into m cells between which the exchange process for k cells can be written in the form

$$\frac{dc_k}{d\tau} = \frac{Q_k^{\text{in}}}{V_k} c_k^{\text{in}} - \frac{Q_k^{\text{out}}}{V_k} c_k^{\text{out}}, \tag{1}$$

where the magnitudes of the discharges through the cell faces are computed by using the hydrodynamic model [4]. When a solid phase is present in the fluid, it will be transported together with the fluid phase in transport motion, additional inertial motion, and also because of mass elimination on the solid–fluid boundary. The first transfer is described by the model (1). The additional transfer is due to the influence of inertial fields and the difference in the phase densities, where the additional flux in the direction of rotational motion of the carrying phase can be neglected for subsonic speeds. The axial velocity of the "additional" particle deposition or levitation motion can be defined [6, 7] as

$$w_{\text{add}}^z = \frac{10^{[-1.82+(1-c)]}\,\text{Ar}\,\mu c^2}{(36+1.22\sqrt{\text{Ar}})\,\rho_p r_p}, \quad c > 0.7, \tag{2}$$

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